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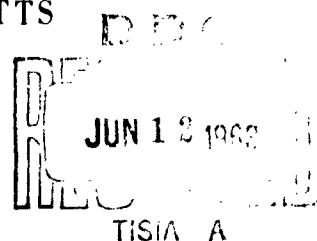
MULTI-REFLECTOR OPTICAL RESONATORS

Peter O. Clark

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Multi-Reflector Optical Resonators

Optical masers with four-mirror cavities have been proposed as gyroscopes and sensitive interferometers¹. A working model of a ring laser rotation rate sensor using four lasers in a four-mirror square cavity has recently been reported². This note describes an analysis based on geometrical optics which determines the conditions for stable operation of such devices. The same procedure may also be applied to the investigation of mode selection in toroidal ruby lasers³ by periodic variations in the reflectivity of the toroid surface.

Investigation of optical resonators other than the two-mirror resonator with plane⁴ or spherical⁵ reflectors, has shown that the geometrical optics of cavities with more than two reflectors can be treated in a manner similar to that of Pierce⁶ and Boyd and Kogelnik⁷. As an example, consider a four-mirror system (see Fig. 1) which uses spherical reflectors 1 and 1' with radius of curvature b_1 and reflectors 2 and 2' with radius of curvature b_2 .

The symmetry of the resonator allows the path of a ray to be described by a set of four difference equations

$$s_n - r_n = r'_n \left[l_2 - r_n \tan\left(\frac{\pi}{4} - \delta\right) - s_n \tan\left(\frac{\pi}{4} + \delta\right) \right] \quad (1a)$$

$$r_{n+1} - s_n = s'_n \left[l_1 + s_n \tan\left(\frac{\pi}{4} + \delta\right) + r_{n+1} \tan\left(\frac{\pi}{4} - \delta\right) \right] \quad (1b)$$

$$r'_n - s'_n = \frac{2s_n}{b_2 \cos\left(\frac{\pi}{4} + \delta\right)} \quad (1c)$$

$$s'_n - r'_{n+1} = \frac{2r_{n+1}}{b_1 \cos\left(\frac{\pi}{4} - \delta\right)} \quad (1d)$$

r_n refers to the height of the ray at mirror 1 or 1' measured perpendicular to the axis, and r'_n refers to the slope of the ray reflected from mirror 1 or 1' measured relative to the axis. s_n and s'_n refer to the corresponding quantities measured at mirror 2 or 2'. The axis of the system is defined as the line joining the centers of consecutive mirrors, and the standard sign conventions of geometrical optics have been used⁸. It should be noted that in order to describe a complete circuit about the resonator the equations must be applied twice.

Exact solutions to the simultaneous nonlinear equations (1a) to (1d) have not been obtained. However, since $l_1, l_2 \gg r_n \tan(\alpha - \delta), s_n \tan(\alpha + \delta)$, a good approximation may be obtained by neglecting these terms in equations (1a) and (1b).

Elimination of three variables leads directly to a linear, homogeneous difference equation for the approximate solution $r_n^{(0)}$.

$$r_{n+2}^{(0)} - l_1 l_2 \left\{ \left[\frac{1}{l_1} + \frac{1}{l_2} - \frac{2}{b_1 \cos(\frac{\pi}{4} - \delta)} \right] \left[\frac{1}{l_1} + \frac{1}{l_2} - \frac{2}{b_2 \cos(\frac{\pi}{4} + \delta)} \right] - \left(\frac{1}{l_1^2} + \frac{1}{l_2^2} \right) \right\} r_{n+1}^{(0)} + r_n^{(0)} = 0. \quad (2)$$

An identical equation for $s_n^{(0)}$ could also be written.

$r^{(0)}$ will remain finite and have the form⁹

$$(A \cos n\theta + B \sin n\theta) \quad (3)$$

provided that

$$0 \leq \left[1 - \frac{l_1}{b_1 \cos(\frac{\pi}{4} - \delta)} \right] \left[1 - \frac{l_2}{b_2 \cos(\frac{\pi}{4} + \delta)} \right] + \left[1 - \frac{l_1}{b_2 \cos(\frac{\pi}{4} + \delta)} \right] \left[1 - \frac{l_2}{b_1 \cos(\frac{\pi}{4} - \delta)} \right] \leq 2 \quad (4)$$

A graphical representation of the stability condition (equation 4) can be easily obtained in two cases:

(a) $l_1 = l_2$

For the rhomboidal resonator equation (4) reduces to

$$0 \leq \left[1 - \frac{l}{b_1 \cos(\frac{\pi}{4} - \delta)} \right] \left[1 - \frac{l}{b_2 \cos(\frac{\pi}{4} + \delta)} \right] \leq 1 \quad (5)$$

The stability condition for the rhomboidal resonator is illustrated in Fig. 2(a).

(b) $b_1 = b_2$ and $\delta = 0$

For the rectangular resonator equation (4) reduces to

$$0 \leq \left[1 - \frac{\sqrt{2} l_1}{b} \right] \left[1 - \frac{\sqrt{2} l_2}{b} \right] \leq 1 \quad (6)$$

Fig. 2(b) illustrates the conditions for stable operation of the rectangular cavity. In Figs. 2(a) and 2(b) the coordinates have been chosen to facilitate comparison with the stability conditions for the two-mirror cavity investigated by Boyd and Kogelnik⁷.

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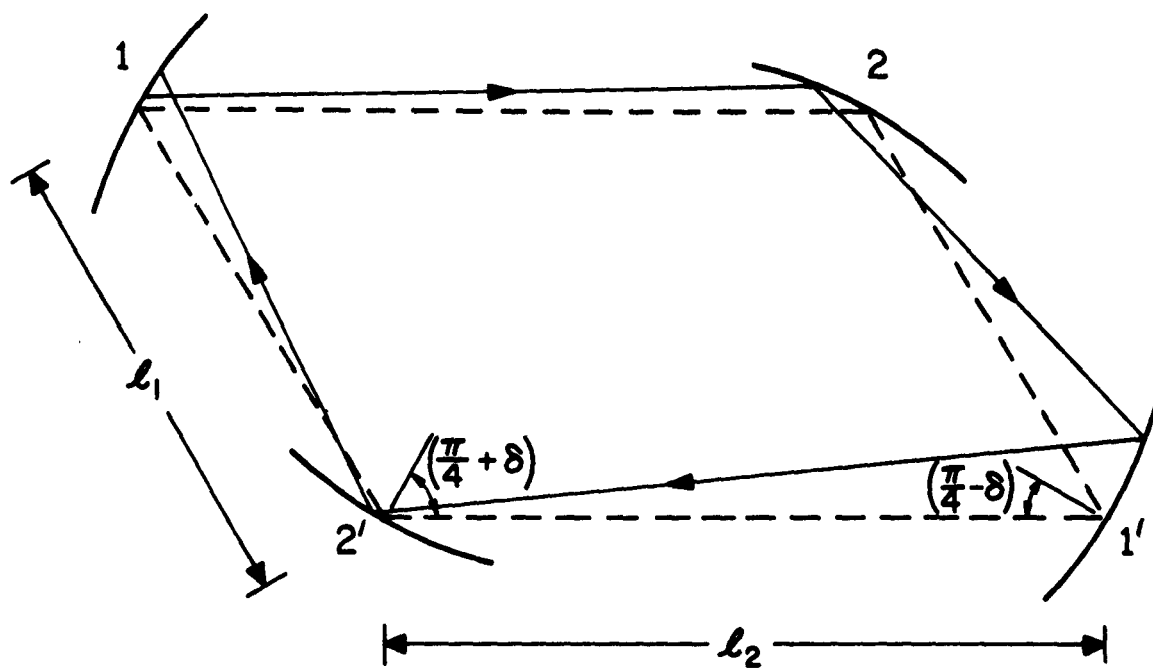


Fig. 1. Four-mirror optical resonator

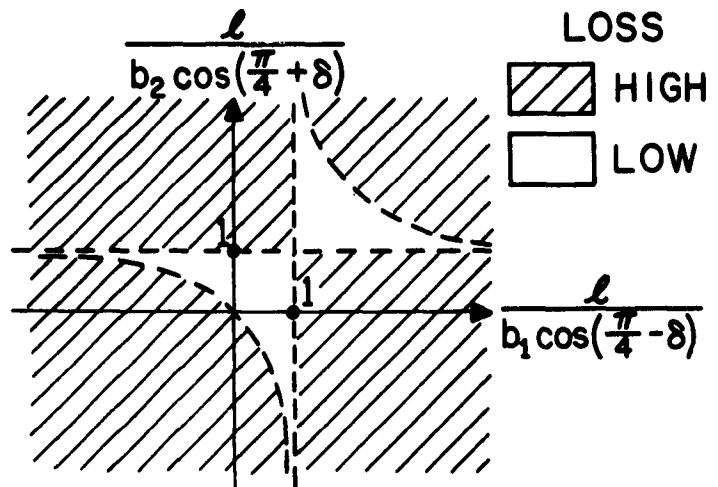


Fig. 2(a). High and low loss regions for the rhomboidal resonator

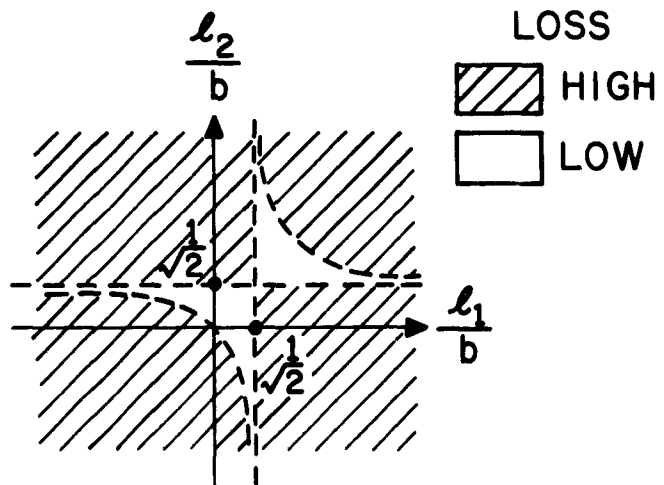


Fig. 2(b). High and low loss regions for the rectangular resonator

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